University of Zurich[UZH]

Analysis of cold based hanging glaciers using a Lagrangian damage model

GEO 511 Master's Thesis

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Abstract:

This master’s thesis focuses on the Analysis of cold-based hanging glaciers using a Lagrangian damage model developed by Mercenier et al. in 2018 (Mercenier et al., 2018). Hanging glaciers are small and found on steep slopes at high altitudes. Hanging glaciers also contain a natural hazard risk because of their avalanching behavior (Margreth et al., 2017). The project aims to establish a better understanding of the dynamic processes of hanging glaciers through modeling. The focus is on answering what sensitivities the model shows to changes in parameters and whether it can estimate how the model compares to a real glacier. The modeling approach consists of a general assessment of the model, a parameter study, a brief investigation of critical stable geometries, and as a final step the representation of a real glacier. The research showed that the model works well and represents glaciers as expected. The parameter study revealed that Glen’s flow factor did not affect stability, decreasing the damage rate and increasing the healing term improves the stability, and that increasing paired slope, slope difference length, and back height decreased stability. The model also displayed that glaciers with varying lengths converge on a critical stable geometry for fixed parameters, which responds to parameter changes as expected after the parameter study. When modeling the simplified Grande Jorasses glacier, the modeling results suggested that either the damage rate used is not suitable and needs to be decreased, the healing plays a very important role, or a combination of both. As the concept of healing does not have the thermodynamical foundation as other parameters (Pralong & Funk, 2005), the damage rate used was calibrated from data from wet calving glaciers (Mercenier et al., 2018). We do not feel confident to judge what is true. But it raises an interesting basis for future research.

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1. Introduction

This master thesis focuses on modeling cold-based hanging glaciers using a Lagrangian damage model developed by Mercenier et al 2019, (Mercenier et al., 2019). The thesis will not only explore hanging glaciers, but we will also focus on how the model works. While hanging glaciers are rather small and might get overlooked, they are fascinating and are important to be researched. Hanging glaciers represent a natural hazard risk, as the ice slabs themselves, but more often secondary triggered avalanches bring a potential danger for life or property. One such example is the hanging glacier located on the West face of the Eiger. Ice slabs and secondary avalanches threaten the Eigergletscher railway station situated at the margin of the assumed flow path, below the glacier, and possible avalanche release areas (Margreth et al. 2017). As hanging glaciers are still not well understood, modeling could help to improve the picture. A possible usage of hanging glacier modeling could be in the integration into existing early warning systems, delivering more insight into what's happening inside the ice. The thesis aims to obtain a better understanding of the dynamics of hanging glaciers through modeling. The main questions are; What are the sensitivities of the model and of hanging glaciers to different parameters? And can it be estimated how good the mode represents real hanging glaciers? We expect that both geometry parameters and non-geometry parameters will be important. For the glacier's stability we think that along with the geometry parameters, the damage rate and healing term will be key parameters. Estimating how close the model comes to reality will be done by modeling a stable geometry of an idealized real hanging glacier and comparing it to the modeled stability. It will then be interesting to see if we can adjust the model accordingly.

1.1. Introduction to cold-based Hanging glaciers

For a better understanding of the topic, we must first take a look at the definition of a hanging glacier, as well as how they form and under what conditions. Hanging glaciers are defined by Pralong & Funk (2006) as "unbalanced avalanching glaciers". An unbalance avalanching glacier is classified as the follows:

"Under constant climatic conditions, these glaciers must release, by calving, a substantial quantity of ice to maintain a finite size geometry. A cycle or pseudo-cycle of icefalls is thus observed. The icefalls recur over a period longer than a hydrologic year, i.e. several years to decades. Therefore, the notion of steady-state geometry does not strictly exist in this context. For avalanching glaciers, the notion of steady-state should be related to an entire icefall cycle." (Pralong & Funk, 2006: 32)

Hanging glaciers, therefore, produce calving, more exactly dry calving. This means that ablation mainly happens through the breaking off of ice slabs without a connection to a water body. In some cases, this can even result in the collapse of the glacier. Important for the definition is also that the bed geometry must allow ice slabs to "fall away from the calving zone". This often means that the glacier bed is located on a slope or at the edge of a cliff (Pralong & Funk, 2006).

Further discrimination is made based on the bed geometry. Pralong & Funk differentiate between terrace glaciers and ramp glaciers. From a two-dimensional perspective from the side of a glacier, the bed of a terrace glacier increases in slope at the glacier margin and may also include an increase in slope at the back of the glacier. On the other hand, ramp glaciers, have no significant bend in their bed geometry. The slope of the bed is in the case of a ramp glacier, the same over the whole length of the glacier (Pralong & Funk, 2006).

The last differentiation in hanging glaciers presented by Pralong & Funk is made based on the calving pattern. Again, taking a look at the glacier from the side in a two-dimensional view, if the calving front is shaped straight from the glacier's surface to the bed then the pattern is called slab fracture. The slab is a block that breaks off at the bedrock. Slab fracture is observed with ramp glaciers, where the slab may represent a large part of the glacier, but also with terrace glaciers, where the slab is the glacier margin. Wedge fracture means that the calving front is curved from the surface towards the front of the terminus and may not reach the bedrock. With wedge fracture, an ice cliff is formed. Wedge fracture is only seen at terrace glaciers where the increase in slope at the glacier margin forms a cliff (Pralong & Funk, 2006).

1.2. Critical geometry

Unbalanced cold hanging glaciers have a critical geometry. This means that for a certain bed topography and climatic conditions, there is a limit on the size and shape of the glaciers before a break-off occurs. While smaller glaciers for the same parameters are stable, with an increase in size and shift in geometry a threshold of stability is reached. If the critical geometry is reached a break-off is unavoidable (Failletaz et al., 2015). Furthermore, for a large-scale break-off event a critical geometry must be reached, however, Vincent et al. have shown that it is not sufficient (Vincent et al., 2015). Even if the critical geometry is reached, the resulting break-off might just be a series of small events (Vincent et al., 2015). In this thesis we will introduce the critical stable geometry, meaning the geometry that is stable after a series of break-off events. The critical stable geometry approaches stability from the other perspective, we are interested in the threshold when stability is reached again.

1.3. Cold based

This thesis is focussing on the analysis of cold-based hanging glaciers. The classical definition of cold-based glaciers says that the basal ice temperature is below the pressure melting point. Cold-based ice is traditional and is also viewed as "frozen to the bed", strongly inhibiting subglacial processes. Most importantly it is considered that there is no basal sliding when a glacier is cold-based. This traditional view has however been challenged and research is focusing on developing a more differentiated approach to cold-based glaciers (Waller, 2001).

1.4. Ice fracture healing

Damage in the ice forms as pressure and stress are forming instabilities and fractures of various scales in the ice. While healing has not yet been researched thoroughly, the concept is not new. On a large-scale glaciers do not only form crevasses, but they can also close these cracks in the material and therefore, re-establish cohesion and stability.

Healing is not new in glacier modeling either, but there isn't a firm grasp on the topic for damage formation for example. Pralong & Funk stated in 2005 that crack healing is a very important process and research needed to focus on developing a physical and thermodynamically founded concept to parameterize healing (Pralong & Funk, 2005).

1.5. Natural hazard risk

As we have already briefly mentioned before, hanging glaciers have the potential to pose a natural hazard risk. Combined snow-ice avalanches resulting from glacier break-offs can endanger life and infrastructure. This is the case with the Whymper Glacier, located at the Grand Jorasses. The Val Ferret and particularly the village of Planpincieux are endangered, and tourism all year round is also affected by the threat. The glacier has had large-scale break-off events in recent history (Margreth et al. 2011). To handle the hazards of hanging glaciers safety concepts are based on calculations and model runs for different scale break-off events (Margreth et al.

2011). Early warning for hanging glacier break-off events is based on the monitoring of the surface displacements. At the Grande Jorasses, the early warning site is based on GPS and close-range photogrammetry (Failletaz et al., 2016). Other early warning systems also rely on seismic measurements or ground-based synthetic aperture radar systems (Vagliasindi et al., 2010).

This concludes the introduction to the topic of hanging glaciers, up next is the methodological chapter, describing the model and our modeling approach.

2. Method

2.1. Model

The model used in this research project is the transient Lagrangian multiphysics ice flow-damage model developed by Mercenier et al. (Mercenier et al., 2018). The model was developed to study calving, exploring the influences of the terminus geometry and water level on the dynamic process, with the goal of better understanding this process through modeling. Similar to the study by Mercenier et al. (Mercenier et al., 2019), we are trying to improve the understanding of hanging glaciers through modeling. The model is suitable to be adapted to hanging glaciers, as we are also interested in calving, however in dry calving, which is rather a simplification of the existing model as the influence of water on the calving process does not need to be modeled. We are also interested in the influence of the glacier's geometry on the stability of hanging glaciers.

First off, the model uses the finite-element library libMesh (Kirk et al., 2006) to implement the stokes equation for incompressible fluid flow with power-law rheology (Glen's flow law) (Mercenier et al., 2020). The following quote from Mercenier et al. summarizes the process of what the model does:

"The computational domain (the glacier) is discretized with a finite-element mesh. The continuum Stokes flow equations with strain-rate-dependent viscosity in an Eulerian reference frame are solved on this mesh at each time step. The resulting velocity field, defined on the mesh nodes, is used to calculate the evolution of a scalar damage variable, and for advection of the mesh nodes. An elimination algorithm deletes mesh elements that are considered destroyed and/or detached, representing glacier calving and oceanic melt." (Mercenier et al., 2019: 3058)

2.1.1. Ice flow and rheology

For each time step, the ice flow is calculated using the Stokes equations for incompressible fluid flow with continuum conservation of momentum and mass. The ice flow is regarded as isothermal and gravity-driven (Mercenier et al., 2018).

2.1.2. Boundary conditions

The glacier's surface is defined as a traction-free surface boundary, this includes the front of the glacier. The glaciers bed is defined through a slipperiness coefficient C, as well as the basal velocity and shear stress. A two-element layer with a constant viscosity is added at the bottom boundary representing a sediment layer (Mercenier et al., 2019). Because the model should represent a cold-based hanging glacier, the bottom boundary is defined as a no-slip boundary. The upstream boundary is defined through a zero-velocity boundary condition (Mercenier et al., 2019).

2.1.3. Damage evolution

The model assumes isotropic damage, which is caused by mesoscale mechanical material degradation. This degradation of the ice's stability is represented by the damage variable D

(Mercenier et al., 2019). The evolving state damage variable D (Pralong & Funk, 2005), which is dependent on the following five variables: the damage rate B a stress measure χ (for which alpha=0.21 and beta=0.63 were chosen following Mercenier et al., 2019), the stress threshold σth (0.11 was chosen following Mercenier et al., 2019), the power r (chosen at 0.43 according to Pralong & Funk, 2005) and the healing term h (Mercenier et al., 2020) which in itself is made up by he0 the initial healing and hsig the healing in relation to stress. D=0, therefore, stands for undamaged ice and D=1 for fully damaged ice. Damage evolution only begins when the stress threshold σth is exceeded (Mercenier et al., 2019).

2.1.4. Geometry evolution

For each time step, the mesh geometry is updated according to the calculated ice flow (Mercenier et al., 2019). The ice flow direction is modeled from left to right, therefore in the direction of positive x coordinates. Additionally, the geometry is updated by the elimination algorithm. This algorithm deletes elements where the value of the damage parameter has exceeded the damage threshold of 0.5, representing ice that has become too weak and lost its stability. The model also checks for elements that are no longer connected to the mesh or the bottom boundary and deletes them (Mercenier et al., 2019). This process is shown in figure 1, where the elements in red have exceeded the damage threshold and are deleted. The elements that were disconnected from the rest of the mesh when the red elements were deleted are eliminated as well, resulting in the updated geometry on the right.

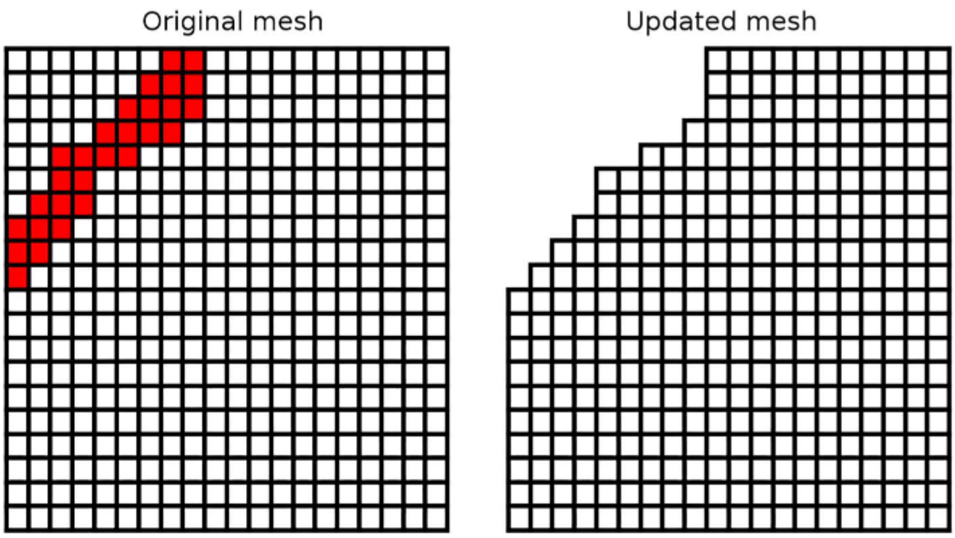


Figure 1: Deletion Algorithm mesh updated: The red squares in the left mesh at time step n have exceeded the damage threshold. The red squares are detected and deleted by the algorithm. Every square that has been separated from the mesh is being deleted as well. This results in the new mesh on the right for time step n+1. Figure 2 in Mercenier et al., 2019

The study of Mercenier et al. showed that the modeled volume loss did not differ substantially depending on the spatial resolution of the mesh, this helps as the computational time for a smaller resolution mesh is lower (Merc 2019).

The model code was adapted to hanging glaciers by removing the feature of a water level, as well as the change in the change in glacier geometry. These changes as well as a refurbishing of the code were performed by Dr. Martin Lüthi. The clean-up of the model code increased clarity and user-friendliness, for example, it was implemented that the destination directory for the model output files can be chosen directly in the model input file, allowing multiple different model runs to be made in a short time and to be conveniently stored in named directories.

2.2. Analysis tools

2.2.1. Cygwin

While Cygwin is not an analysis tool, it was a crucial software for the project as it allows for remote access to Linux based server on which all the modeling was calculated, from a Microsoft windows-based client. Cygwin is an open-source collection of Unix tools and made this thesis possible.

2.2.2. ParaView

For the analysis of the model runs the ParaView 5.10.0 RC-1 software was used. ParaView is an open-source free-to-use software platform for the analysis and visualization of data, developed by Kitware. The software is supported on multiple platforms which were necessary as the model is based on a Linux environment, the analysis of the data was however conducted on a Microsoft Windows operating system for logistical and practical reasons. ParaView allows for visualization of the output files of the model runs. The data can be analyzed per time step or as a sequence. It can directly display values of different variables, for example, the damage variable as a color code on the mesh. The software also allows to extract data, for example, damage along a profile line, and display the graph. To get an impression of the software's capabilities figure 2 displays the user interface for the aforementioned examples.

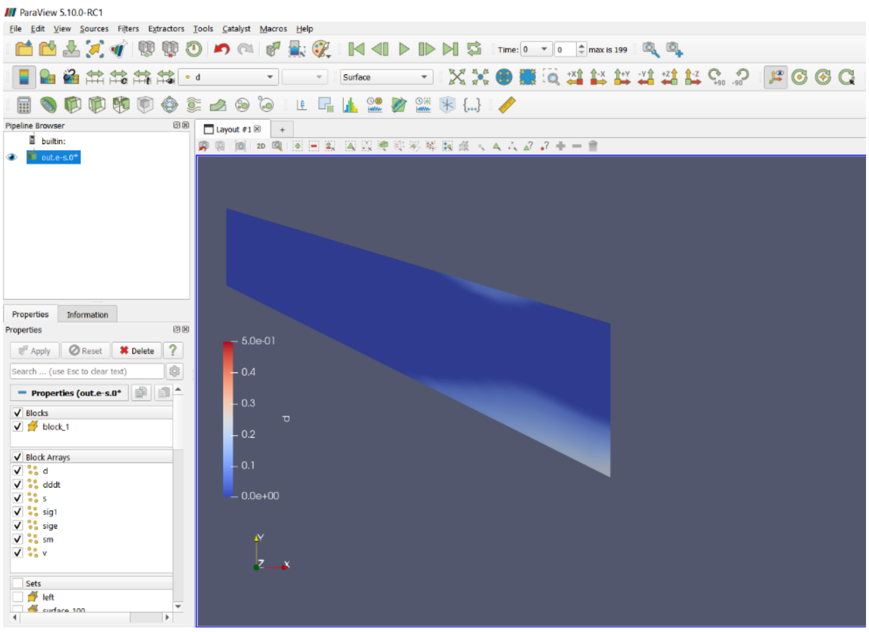


Figure 2: General overview of the ParaView software user interface.

2.2.3. Python pre-loop

For the parameter study, two python pre-loops were used, which were written by Dr. Martin Lüthi. The first pre-loop created input files for a combination in the ranges of two parameters, for example, the surface slope and the bed slop. These input files were placed in named directories according to the input parameters of interest. For example, for a model run investigating the geometry of a glacier and input parameters of bed slope of x and surface slope of x, the directory "surf_x_bed_x" was created. The second pre-loop executed the model runs in the directories with the associated input file. This helped to generate a lot of data in a short amount of time. The pre-loops also decreased the time needed for manually setting up the model runs.

2.2.4. Python data extract

To extract data from the output files Dr. Martin Lüthi provided two python scripts, which helped to automatically extract and plot data. These python scripts were mainly used to extract bed stress data and plot their variation depending on the parameter values.

2.2.5. Excel

Excel was used for plotting the break-off rates for the different parameter variations. Excel was chosen as it offers a straightforward platform for generating value tables and subsequently plotting these values. It was easy to use as most people are probably going to be at least somewhat familiar with Excel.

2.3. Modelling Approach

We applied the Lagrangian damage model developed by Mercenier et al. for a two-dimensional space, which helps to keep computational times low (Mercenier et al., 2018). The terminus front was chosen to be vertical to match the typical shape of hanging glaciers. We also did neglect accumulation to keep the model as simple as possible.

At the beginning of the project, some time was needed to get used to the Linux environment, as this was completely new and quite different in function to more common operating systems used, like Microsoft Windows or macOS. The first modeling steps were to play around with different parameter combinations to get a feeling for how to model works, what it is capable of and what would make sense in investigating, generally speaking getting to know the model. After this first phase, a very simple and small-scale parameter study on the combinations of surface and bed slopes was conducted as a preparation for the large-scale parameter study involving more variables later on. During this phase regular meetings with Dr. Martin Lüthi helped for a better understanding of the model and his feedback improved the workflow. He also provided help and support when questions arose or when there were adjustments to the model code needed. This resulted in the refurbished code and later in the pre-loops and python analysis scripts. At the end of this first modeling phase, a first general assessment of the model was made. The goal was to identify the

characteristics of hanging glaciers that were displayed by the model. This first evaluation of the model was rather general and purely qualitative, but also involved looking at data gathered on real hanging glaciers and trying to find similarities. This first modeling phase laid the foundation for the parameter study and the parameter refinements for the modeling of real hanging glaciers.

2.3.1. Parameter study

The first part of the research project is the parameter study, which was conducted to explore the sensitivity of the model to the various parameters. As a basis for this analysis, a standard glacier geometry for reference was used. This idealized glacier had the following geometry: 100m length, 20m back height, 0.3 surface slope, and 0.5 bed slope (slopes are given as the sinus of the angle). For the other parameters that do not describe the glacier's geometry, standard values were set according to values that were already set in the model provided to me and discussed with Dr. Martin Lüthi. The idealized glacier was discretized as a mesh of 20 vertical and 20 horizontal elements. For the time stepping, we chose 0.01 and 200 time steps, which results in a period of around two years. Table 1 provides an overview of the parameters that were explored in the parameter studies, the standard values. The parameters for surface and bed slope, as well as the healing parameters, were both examined for themselves as a combination of the two-parameter pairs to take a closer look at how they are interlinked and working together. Unless differently stated this parameterization and values were used as a basis/reference.

Table 1: Table of parameters and their respective standard values that were explored in the parameter study

Variable name	Standard value	Source/comment
Damage rate B	65	Mercenier et al. 2018
Glen's flow factor A	30	For ice -5°C
hsig	5	Healing multiplied with compressive stress
he0	0.5	Constant healing
Surface slope	0.3	
Bed slope	0.5	
Length	100	
Back height H0	20	

2.3.2. Critical stable geometry

To investigate the critical stable geometries a glacier standard slope and a back height of 10m were used. Three different lengths were modeled: 80m, 100m, and 140m. To keep the horizontal resolution the same the number of horizontal elements was adjusted, 16 elements for 80m, 20 for 100m, and 28 for 140m. This way the horizontal elements kept their size and were, therefore, more easily comparable. To see how the critical stable geometry shifts when a geometry parameter is varied, a thinner H0=5m glacier was created. This also allowed us to see if the critical stable geometry would react as expected from the results of the parameter study.

2.3.3. Real glacier modeling

As an approach to verification of the model, we used a simplified geometry of the Whymper glacier located at the Grande Jorasses. The simplified geometry was derived from the glacier profile shown in Figure 12 in Pralong & Funk (Pralong & Funk, 2006). The exact geometry was the following: a surface slope of 0.6, a bed slope of 0.7, a back height of 10m, and a length of 180m. This is treated as the critical stable geometry so for the model run, we extended the length to 200m, a supercritical value. We then conducted a model series consisting of a range of damage rates from 2-65, for two coupled healing terms, once for hsig=5 and he0=0.5 and once with hsig=15 and he0=1.5. We made all this run once with A=75 and once with A=30 to see if it might make a difference on such a step, thin glacier.

3. Results

3.1. General overview

We will start with a general assessment of the model by looking at the model output for the standard glacier geometry with standard parameters. This first assessment consists of purely qualitative observations and helps to improve our understanding of the model before going over to the parameter study and the consequent modeling steps. All general observations were made using the standard geometry glacier and standard parameters as described in chapter 2, table 1.

3.1.1. Damage pattern and evolution

The model output for the standard glacier geometry with standard parameters is displayed in figure 3. On the left is the image for the time step 0, and the right-hand image shows time step 2. The color code shows the values for the damage variable in the range of 0 (in blue) up to 0.5 (in red). It is visible that damage evolves in two concentrated areas. The first damage zone is at the base of the glacier's terminus, reaching back to approximately half the length of the glacier and about a third of the glacier's thickness at the terminus. The upper edge of the damage zone is parallel to the bed starting at the terminus, but switches to the horizontal after the first quarter of the length, resulting in a trapezoid shape. The second damage zone forms at the glacier's surface, again at around a quarter in length from the terminus. The shape of this second damage form describes a shallow half oval. This second zone is much smaller than the first damage zone and lower damage values are reached. In the first zone, damage values reach over 0.2, while values in the second zone do not exceed 0.2 at time step 0. At time step 2 we can see that damage values at the base of the first damage zone have already reached just under 0.5, which will result in the deletion of the elements in the next time step. In damage zone two we do not come close to reaching 0.5, but the damage will persist as we can see later in figure 5.

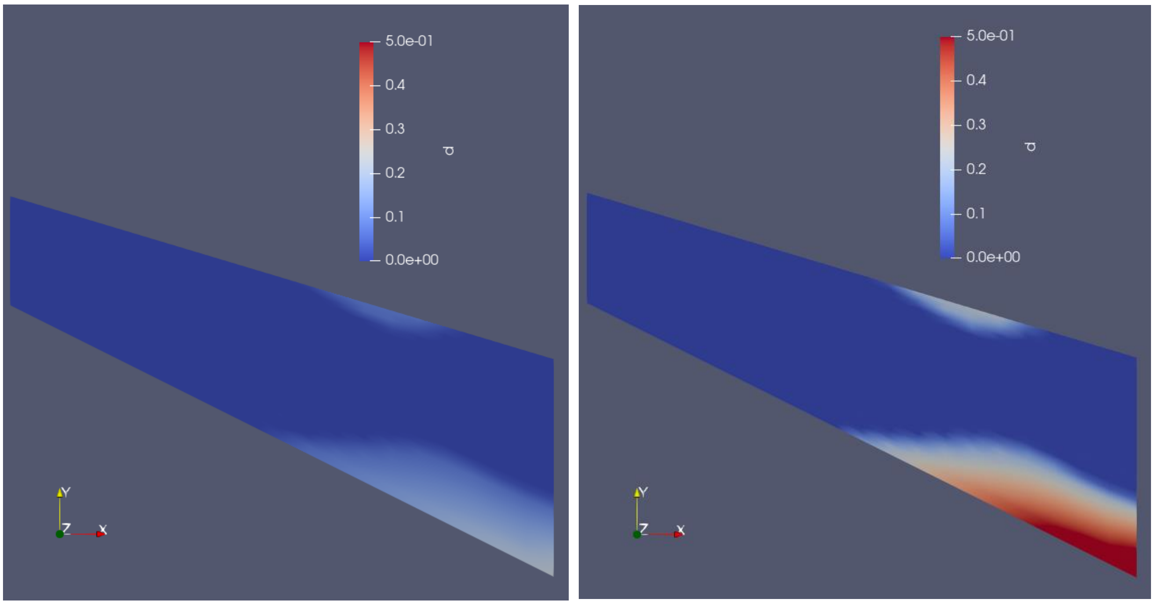


Figure 3: Side by side comparison of the standard geometry glacier for time step 0 (left) and time step 2 (right). The color code displays the damage values from 0 to 0.5.

Figure 4 shows the damage evolution with time (dD/dt) for the same model output as in figure 3. From left to right we see the time steps 0, 1, and 5. Again the range of damage evolution is set from 0 to 0.5. In this figure, we can see that the evolution of damage is higher in the first damage zone at the terminus base. In both zones, the values increase from time step 0 to time step 1 and both zones seem to grow in size. From time step 1 to time step 5 however, damage evolution has slowed down in the first zone and the second damage zone has completely vanished.

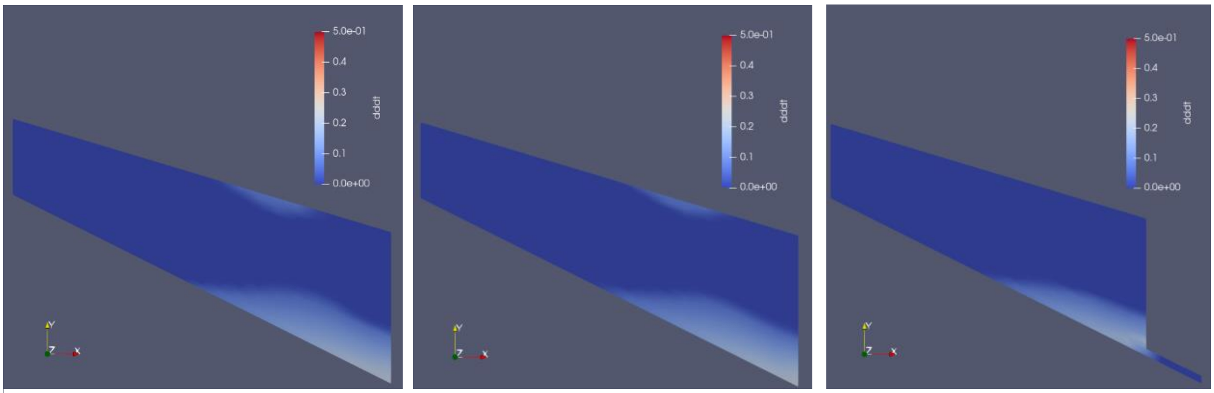


Figure 4: Damage evolution over time (dD/dt) as color code (value range 0-0.5) for the standard glacier geometry at the time steps 1 (left), 2 (center) and 5 (right)

3.1.2. Break-off pattern

The model displays a slab break-off pattern, with the principal failure being the first damage zone at the terminus base of the glacier. Beginning at time step 3 the glacier begins calving one element per time step. Figure 5 shows the modeled damage evolution distribution of the standard glacier geometry at time step 8. We can see that both damage zones still exist, however, only damage zone one is moving with the glacier retreat. Damage zone two at the surface stays in place and will later vanish as the glacier retreats further. Another observation is that the base damage zone has transformed its shape and consists now of a triangle between the bed, glacier front, and a horizontal line. With the glacier retreat, the first damage zone is shrinking. After time step 13 the break-off rate seems to gradually decrease as the number of time steps an element survives increases. This is also mirrored in a gradual decline in values for the damage evolution over time.

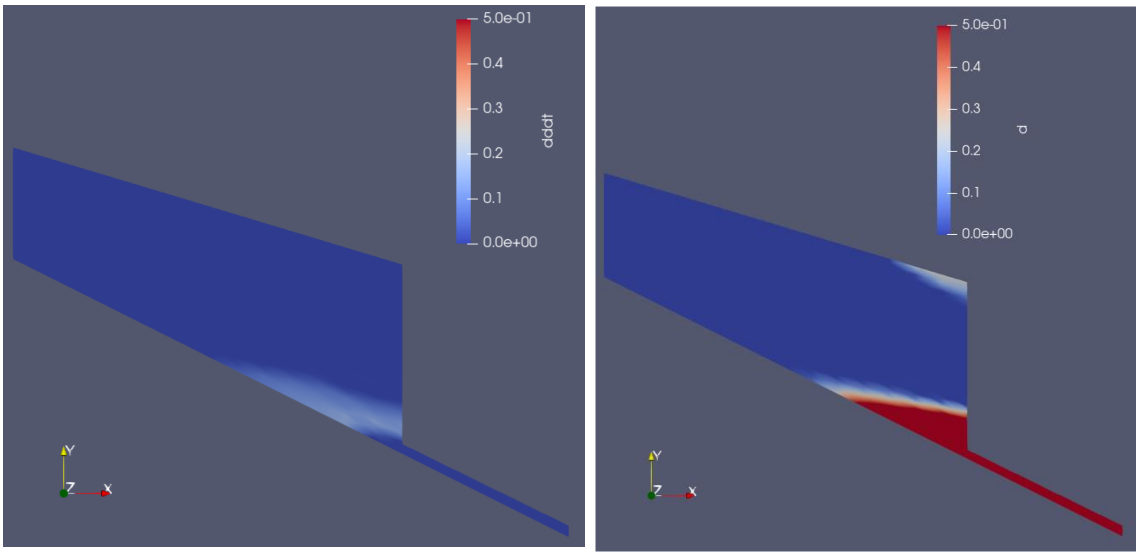


Figure 5: Damage evolution over time (dD/dt) as color-coded on the left, Damage on the right (value range 0-0.5) for the standard glacier geometry at the time step 8

3.1.3. Additional observations

3.1.3.1. Velocity

The observations on the ice flow velocity for the standard glacier geometry at time step 1 (left) and 6 (right) are shown in figure 6. At time step 1 we can see that the ice flow accelerates towards the terminus over the whole thickness of the modeled glacier. However, we can see that the acceleration is decreasing from the surface towards the bed. The acceleration also mainly affects the terminus. At time step 6 the above-mentioned observations are still true, but the

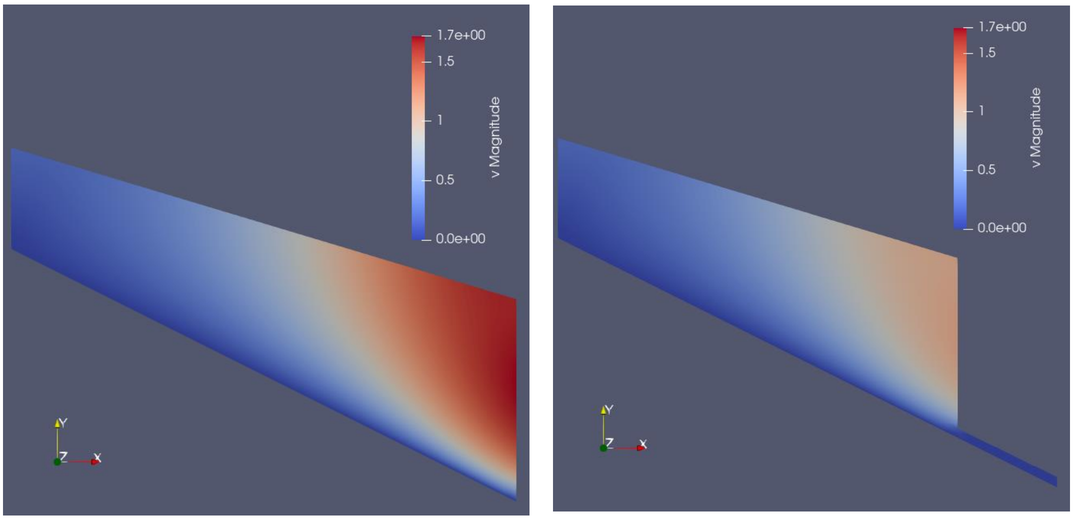


Figure 6: Ice flow velocity distribution color-coded in (blue slow, red fast) for the standard glacier geometry at the time steps 1 (left) and 6 (right)

velocities have strongly decreased.

3.1.3.2. Deformation

While the model does calculate ice flow and deformation, it is not visible in this model output. On one hand, we have a very quick and complete break-off which makes it difficult to observe deformation. On the other hand, the modeled ice is cold for the whole glacier, as the standard parameters include AGlen = 30 for ice that has a temperature of around -5° Celsius.

3.2. Parameter study

The results from the parameter study will be assessed from two perspectives with the first being a more quantitative perspective, as we will take a look at the surface and basal stresses. We also introduce the break-off rate, which describes the gradual retreat of the glacier as dry calving is happening in comparison to the time steps. The second is again the more qualitative perspective we already used for the general assessment of the model, looking at damage patterns and evolution as well as break-off patterns and additional observations. The parameter study addresses the following non-geometry parameters: damage rate (B), the Glen's flow factor (A), and the healing term (hsig and he=). Additionally, the geometry parameters are explored, including the back height (H0), the surface slope, the bed slope, and the length. For all assessments, the standard geometry glacier was used as a basis upon single or pairs of parameters were changed for analysis.

3.2.1. Non-geometry parameters

3.2.1.1. Damage rate

3.2.1.1.1. Damage pattern and evolution

Figure 7 shows the comparison of model runs performed with the damage rate set at 65 (left) and at 10 (right) at time step 2. The damage pattern is the same for both damage rates, it is the typical pattern of two damage areas as described in the general model assessment. Compared to the larger damage rate, the damage areas for the smaller B are smaller in size and the values reached for damage are much lower. While for B=65 the basal terminus damage zone is approaching 0.5, for a damage rate of 10 the damage values do not exceed 0.2.

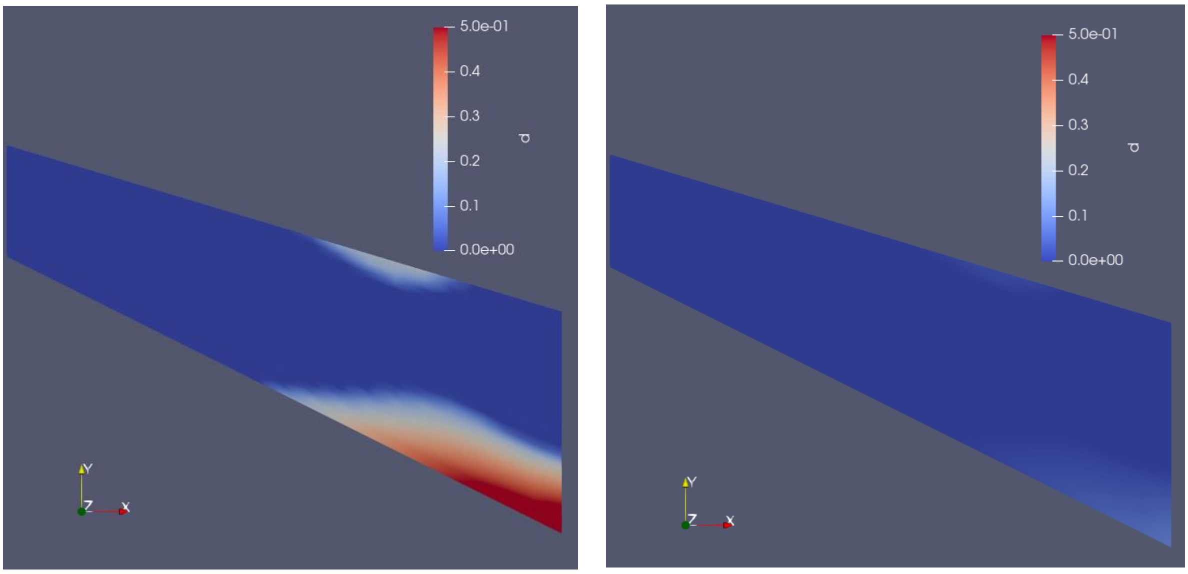


Figure 7: Damage as color-coded for the standard glacier geometry at the time step 2. Damage rate B= 65 on the left, B=10 on the right

Taking a look at figure 8, which shows the damage evolution for the same model runs, we can see a similar effect as in the damage. To be able to see the two damage zones for the model output calculated with B=10 we need to adjust the color code. As we would expect, decreasing the damage rate has tremendously slowed down damage evolution and therefore increases the stability of the modeled glacier.

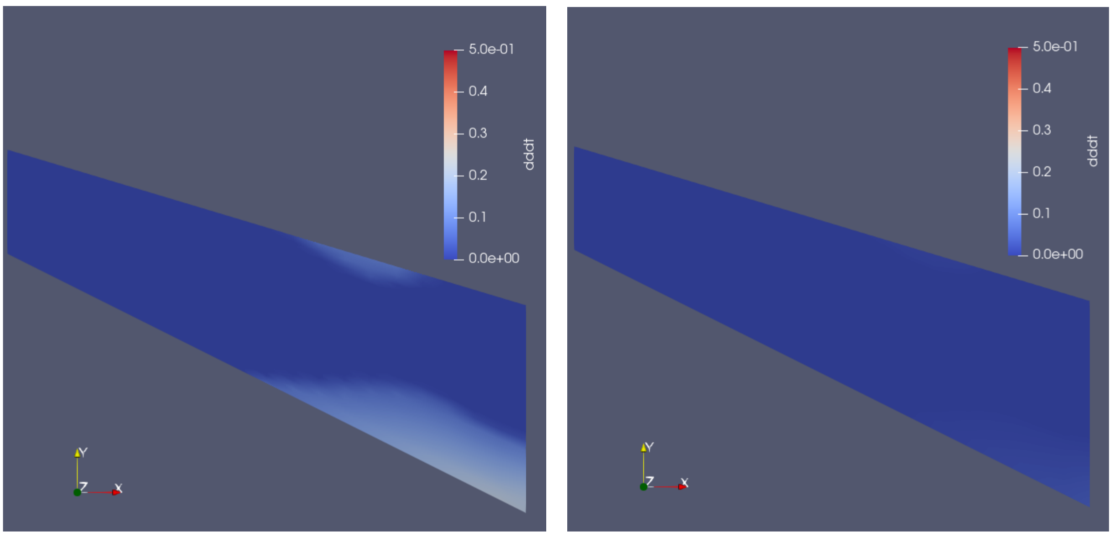


Figure 8: Damage evolution over time (dD/dt) as color-coded for the standard glacier geometry at the time step 2. Damage rate B= 65 on the left, B=10 on the right

3.2.1.1.2. Break-off pattern and rate

Break-offs follow the same pattern for both damage rates and are similar as described in the general model assessment, however with the reduced damage rate the, break-off rate is strongly reduced. Figure 9 shows the plot of the remaining horizontal elements (y-axis) over the time steps (x-axis) after the first break-off event. The figure shows the results for a modeled glacier for a damage rate in 10, 40, and 65. We can see that generally, while the break-off rate is initially high, it decreases over time and approaches a stable geometry. For the different damage rates, we can see that the initial break-off happens much later for a small damage rate and the break-off rate is decreasing and approaching a stable geometry faster. Again, we see clearly that the reduced damage rate leads, as it should, to more stability as damage evolves slower.

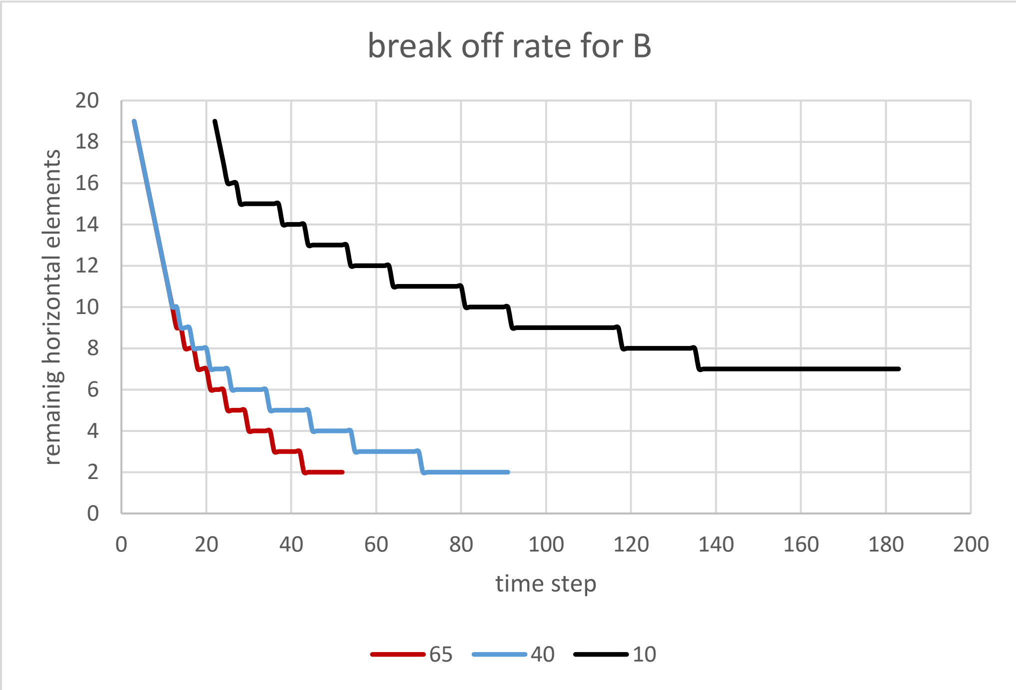


Figure 9: Plot of the break-off rates for the different damage rates (B= 64 in red, B=40 in blue, and B=10 in black) for the standard glacier geometry

3.2.1.1.3. Additional observations

There are two main additional observations: The damage rate does not influence the ice flow velocity. The distribution of velocity is the same for both damage rates. Secondly, at a damage rate of 10, we can see some slight deformation in the ice. With a reduced damage rate, the glacier ice does not degrade and break off as quickly, it, therefore, has time to deform under stress.

3.2.1.2. Glen's flow Factor

For the standard parameters and standard geometry glacier, the change in A from 30 to 75 resulted in no visible differences in damage patterns or evolution. The break-off pattern, as well as the break-off rate, seem to be identical. In the last step of the modeling work, it was discovered that for a low damage rate B the change in A can result in a marginal difference in model outputs. The coupling of the damage rate and Glen's flow factor will not be investigated further, but the marginal finding will be presented in the results of the last step of modeling.

The reason why the Glen's factor does not play a significant role in our modeling experiments, maybe that we are mainly modeling rather thin glaciers, so the overburdened weight does not induce significant deformation. Additionally, we are neglecting accumulation which would induce more overburden pressure and ice flow.

3.2.1.3. Healing

The healing term (h) is part of the damage equation and is split into two parts, the hsig (healing rate multiplied with compressive stress) and he0 (constant healing rate). Both parameters will be addressed separately and coupled together.

3.2.1.3.1. Damage pattern and evolution

The top left image in figure 10 is the reference model output the top right shows the results for hsig= 50 and he0 set a zero. The bottom displays the model run for hsig =0 and he0 = 5 and the bott left shows the coupled healing term with hsig =50 and he0=5. For an increased hsig and he0 of zero, the damage pattern is roughly the same as for the reference model output with standard parameters. However, the terminus base damage zone is much smaller and the core of high damage values has more of a triangular shape than the overall trapezoid. It is also worth noticing that the values for damage in the surface damage zone are higher compared to the reference model results, as well as the whole damage zone is slightly larger. For hsig 0 and he0=5, the damage pattern is again very similar to the reference results. The damage zones are smaller but reach similar values in damage. For the coupling of increased hsig and he0, the results show a drastic decrease in the size of the damage zones. The terminus base damage zone has now taken on an overall rather triangular shape. Damage evolution is reduced and follows the same pattern changes as already discovered in the damage pattern. As an overall observation, we can say that the coupling of the two healing parameters results in the highest reduction of damage compared to the reference model results.

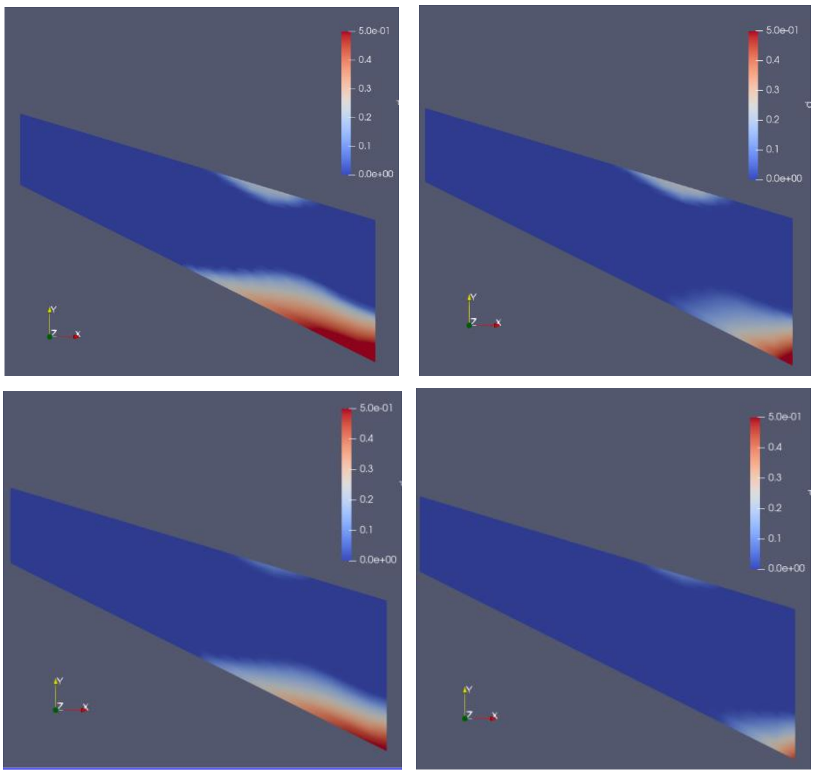


Figure 10: Damage color-coded for different healing terms: top-left reference glacier, top right: hsig=50, he=0, bottom left hsig=0, he0=5, bottom left hsig=50, he0=5 on the standard glacier geometry at time step 2

3.2.1.3.2. Break-off pattern and rate

For the break-off pattern and break-off rate, we will focus on the coupled healing term. The break-off pattern is the same slab break-off pattern as observed with the reference model results. Figure 11 shows the changes in break-off rate with and without coupled healing. As can be seen, the coupled healing decreases the break-off rate slightly and leads toward a stable geometry. The damage evolution is slowed down by the coupled healing the stability increases. This is what we would expect as the healing of fractures re-establishes cohesion and stability in the ice.

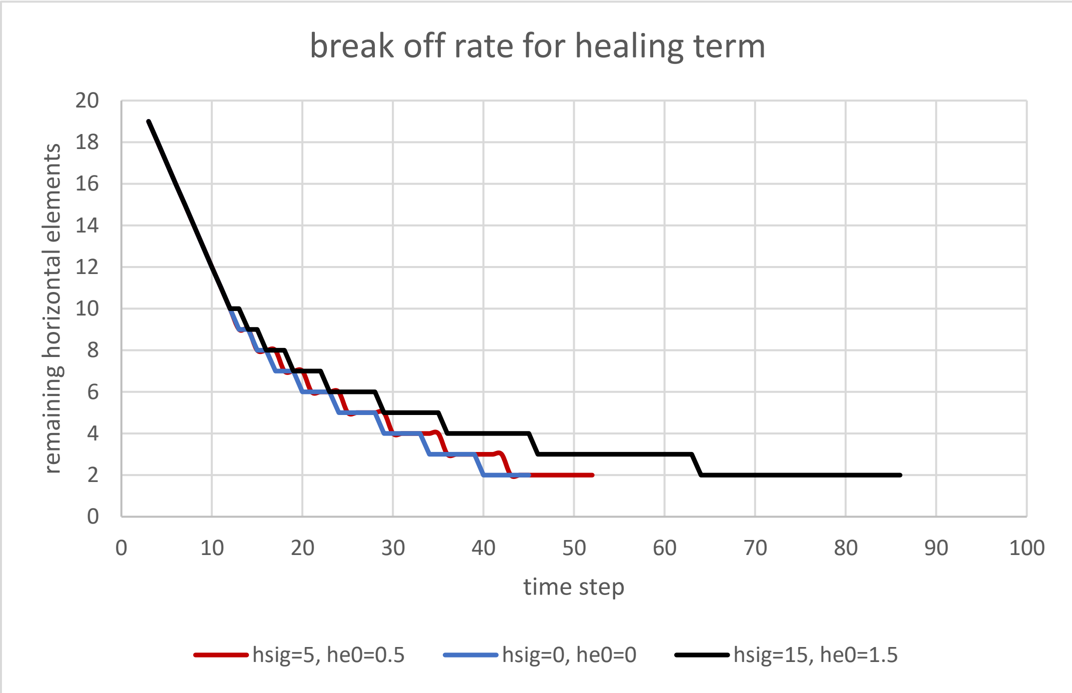


Figure 11: Plot of the break-off rates for the different coupled healing terms (hsig=5, he0=0.5 in red, hsig=0, he0=0 in blue, and hsig=15, he0=1.5 in black) for the standard glacier geometry

3.2.1.3.3. Additional observations

The ice flow velocity distribution does not change with any form of increased healing. This does not mean that the ice flow remains unaffected. With the slowing of damage evolution in the ice, we can again observe slight deformation of the glaciers geometry, similarly to what has already been seen.

3.2.2. Geometry parameters

3.2.2.1. Slope

For the bed and surface slope of the glacier, we will differentiate between the overall slope where bed slope and surface slope are coupled as a pair with a difference of 0.2, for example, a surface slope of 0.1 is coupled to the bed slope of 0.3. Additionally, we will take a look at the influence of the difference between surface and bed slope. It must be acknowledged that when we look at increasing the difference between the surface slopes, the mean thickness is increased as well.

3.2.2.1.1. Damage pattern and evolution

Figure 12 shows two coupled slope pairs, on the modeled glacier on the left has a bed slope of 0.5 and a surface slope of 0.3, the one on the right 0.7 and 0.5 respectively at time step 2. While the flatter glacier shows the typical pattern of damage distribution, the steeper one is missing the surface damage zone and has a prolonged basal damaged zone which has a triangular shape and reaches almost to the back of the glacier. The damage evolution follows the same pattern as already seen in the general model assessment.

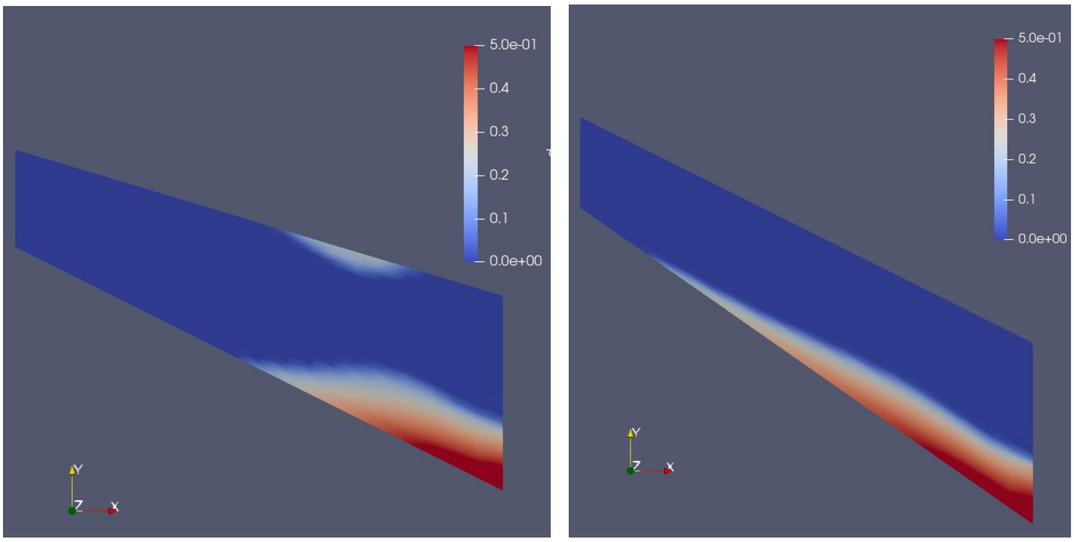


Figure 12: Damage color-coded for different paired slopes: left s=0.3,b=0.5, right s=0.5,b=0.7 at time step 2 (value range 0 (blue)- 0.5(red))

Fig 13 shows the damage pattern for an increasing slope difference between the surface and the bed. On the left image, the difference in slope is only 0.1 and we see only a small damage zone at the terminus. The results in the middle show a 0.2 difference in slope, and we can see the typical damage pattern of the two zones, however, these zones are shaped differently and are converging. For the 0.3 difference in slope, the damage pattern has changed to a large damage zone which has merged the surface and basal damage zone.

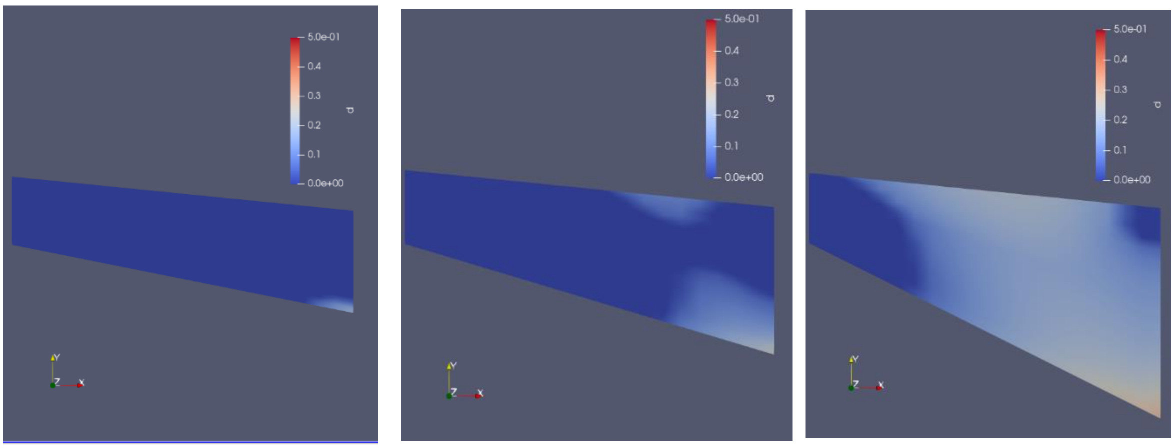


Figure 13: Damage colour coded for three slope differences: left 0.1, middle 0.2 and right 0.4 at time step 2, (value range 0 (blue)- 0.5(red))

3.2.2.1.2. Break-off pattern and rate

For the paired slope glaciers, the break-off pattern is the same as already previously described. Taking a look at the plot of the break-off rates of three different paired slope glaciers shown in figure 14, we can see that steeper glaciers have a higher break-off rate than flatter glaciers. The flatter glaciers are also more likely to end in a stable geometry.

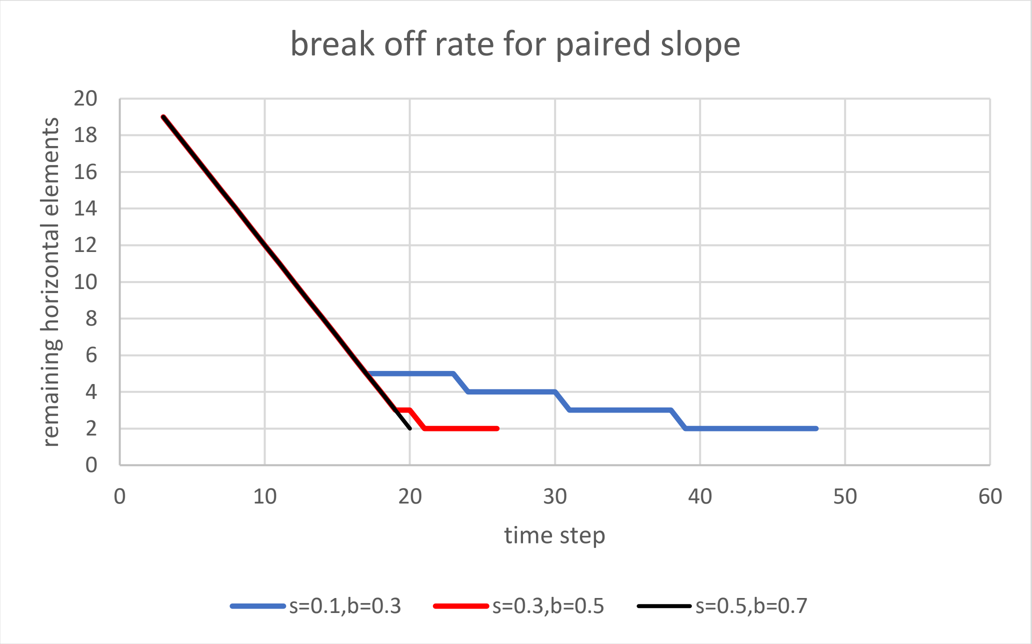


Figure 14: Plot of the break-off rates for the paired slopes (s=0.3,b=0.5 in red, s=0.1,b=0.3 in blue, and s=0.5,b=0.7 in black) for the standard glacier geometry

For the increasing slope difference results, the break-off pattern is similar to the reference glacier results. The break-off rate increases with the increasing slope difference between surface and bed. Figure 15 shows the break-off rates for the same model runs for which we already showed the damage patterns in figure 13.

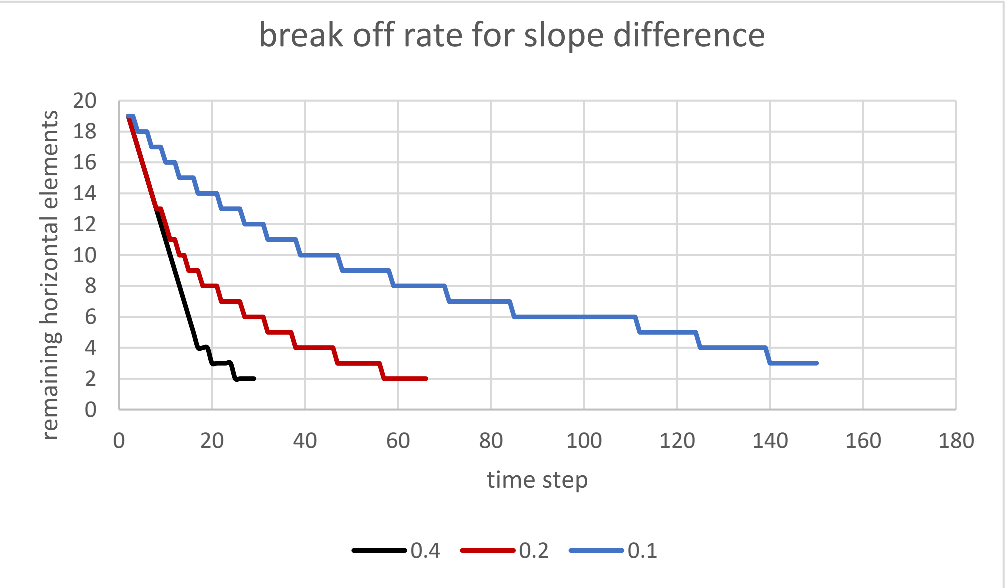


Figure 15: Plot of the break-off rates for the slope difference (0.2 in red, 0.1 in blue, and 0.4 in black) for the standard glacier geometry

3.2.2.1.3. Additional observations

For the paired slope model runs the velocity pattern is very similar to the reference model results. However, we can see that with increasing steepness the zone of higher velocity ice flow is enlarged and reaches further back at the glacier. For the increasing slope difference, the magnitude of the ice flow velocity increases with increasing slope difference.

3.2.2.1.4. Stress analysis

For the paired slope variation, the stress analysis showed an increase in horizontal stress and a decrease in shear stress for an increased slope. The vertical stress showed no significant change. The increase in horizontal stress would therefore be probably the main driver behind faster damage evolution. The stress analysis results can be seen in figure 16.

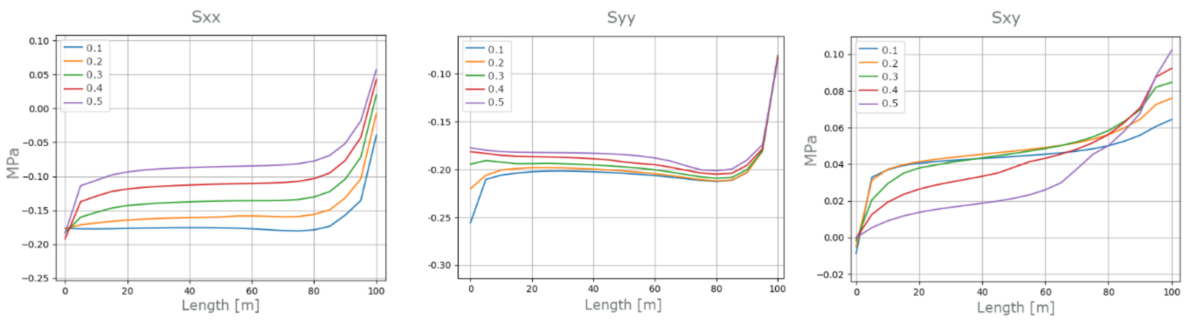
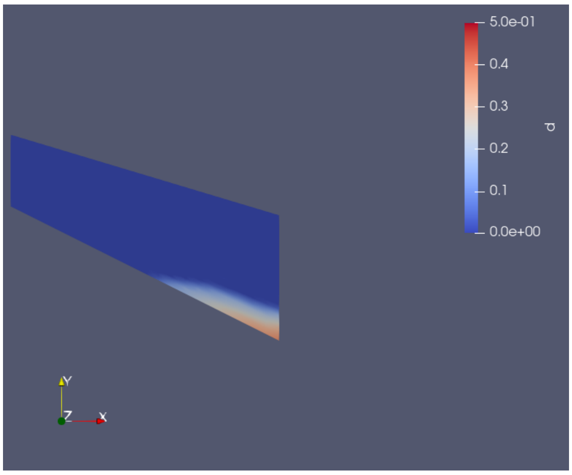


Figure 16: Results of the stress analysis for the paired slopes for surface slopes from 0.1 to 0.5, the respective bed slopes would be 0.3-0.7. The right is sxx (horizontal stress) in the middle syy(vertical stress) and on the right sxy (shear stress)

3.2.2.2. Length

3.2.2.2.1. Damage pattern and evolution

For exploring the length, we chose to perform the modeling with a thinner geometry so that a stable geometry could be reached, letting us get a first glimpse at modeling critical stable glacier geometries. For this reason, the back height was lowered to 10m. We have to address that with an increase in length, again the mean thickness of a glacier, which does not have parallel bed and surface slopes, does increase. Increasing the mean thickness of the glacier is further explored with the parameter for the back height. For a longer glacier, as shown in the right image of figure 17, the damage pattern changes significantly as the two previously described damage zone merges into one large anvil-shaped area. Compared to the standard glacier geometry results, the damage values increase especially at the surface of the glacier. For a shorter glacier as can be seen in the left image of figure 17 the surface damage zone is non-existent and the basal damage zone is much smaller. The damage evolution is distributed in the same pattern as for the damage. With the retreat of the glacier, the damage evolution zones become disconnected, and the surface zone vanishes as the ice breaks off, while the basal zone travels with the glacier as it retreats, as has been seen in the general assessment of the model. As a result, the damage zones follow the same pattern. In contrast to the other model runs performed, resulting from the lower mean thickness, even the basal damage evolution zone vanishes. Because of this a stable geometry forms. This is shown in figure 18.



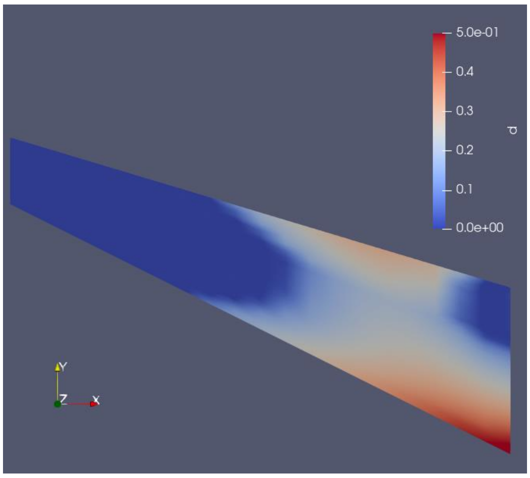


Figure 117: Damage color-coded for 2 different lengths, left 75m,and right 150m (value range 0 (blue)- 0.5 (red)) at time step 2

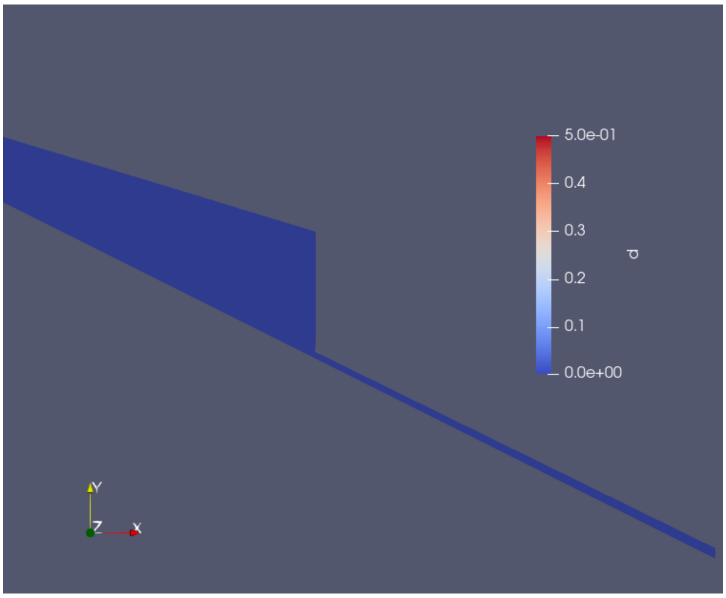


Figure 18: critical stable geometry at time step 199

3.2.2.2.2. Break-off pattern and rate

The break-off pattern is the same as observed previously. However, due to the changes in geometry the glacier breaks off until it reaches a stable geometry. The plot of the break-off rate is very interesting and is shown in figure 19. All rate lines start at a different position, as the remaining horizontal elements vary. The time of the first break-off varies as well. The main point however is that all three converge on the same stable length. This suggests that there is one stable length for the other persisting factors, be it geometry parameters or non-geometry parameters. It is also clear that the shortest glacier is all in all the most stable.

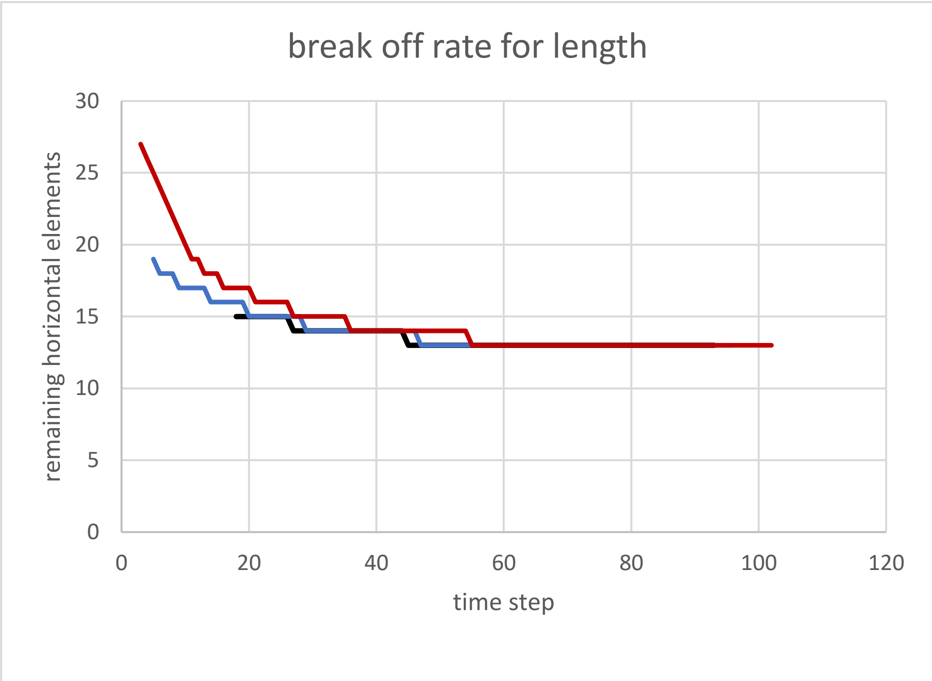


Figure 19: Plot of the break-off rates for different lengths (140m in red, 100m in blue, and 80m in black) for the standard glacier geometry

3.2.2.2.3. Additional observations

While the pattern of the ice flow velocity distribution does not change, the magnitude does strongly differ. For the longer glacier, significantly higher velocities at the terminus are reached. Deformation is only slightly visible with the shortest glacier, it is even less pronounced than what has already been seen before.

3.2.2.2.4. Stress analysis

For the length parameter, the stress analysis showed that increasing length also increases shear, horizontal and vertical stress. As we would expect with the longer glacier there is also a shift of the peak in the stresses. The increased stress explains why increasing length damage develops faster and leads to instability. The results of the stress analysis are shown in figure 20.

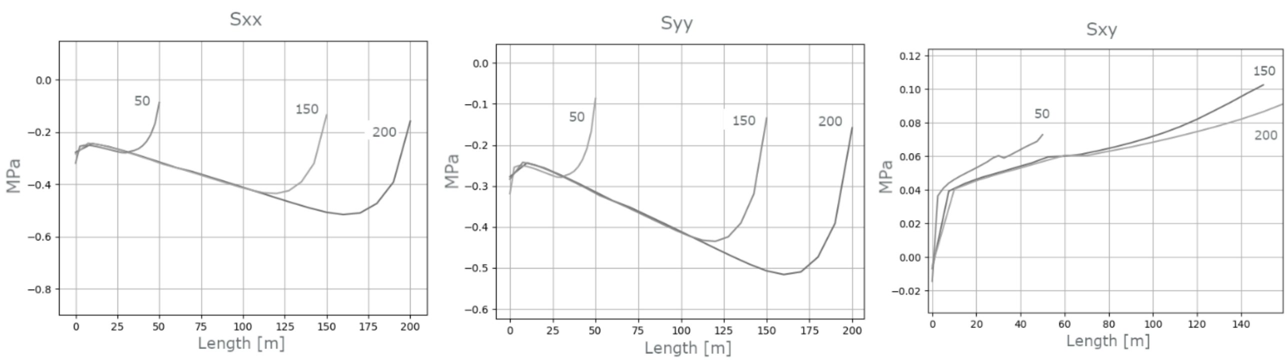


Figure 20: Results of the stress analysis for the different lengths (50m, 150m, 200m). The right is sxx (horizontal stress) in the middle syy(vertical stress) and on the right sxy (shear stress)

3.2.2.3. Back height/mean thickness

3.2.2.3.1. Damage pattern and evolution

As has already been described before the increase in the back height increases the mean thickness. This leads to major changes in the damage pattern. Figure 21 shows the standard reference glacier model with a back height of 10m on the left and 35m on the right. While the thin glacier on the left has a very small concentrated basal damage zone at the terminus, the thicker glacier has only two spots where damage has not set in. This is at the surface of the terminus and the very back basal zone. The damage very quickly spreads over the whole glacier. The thin glacier only develops damage at the base as the basal damage zone travels as the glacier breaks off.

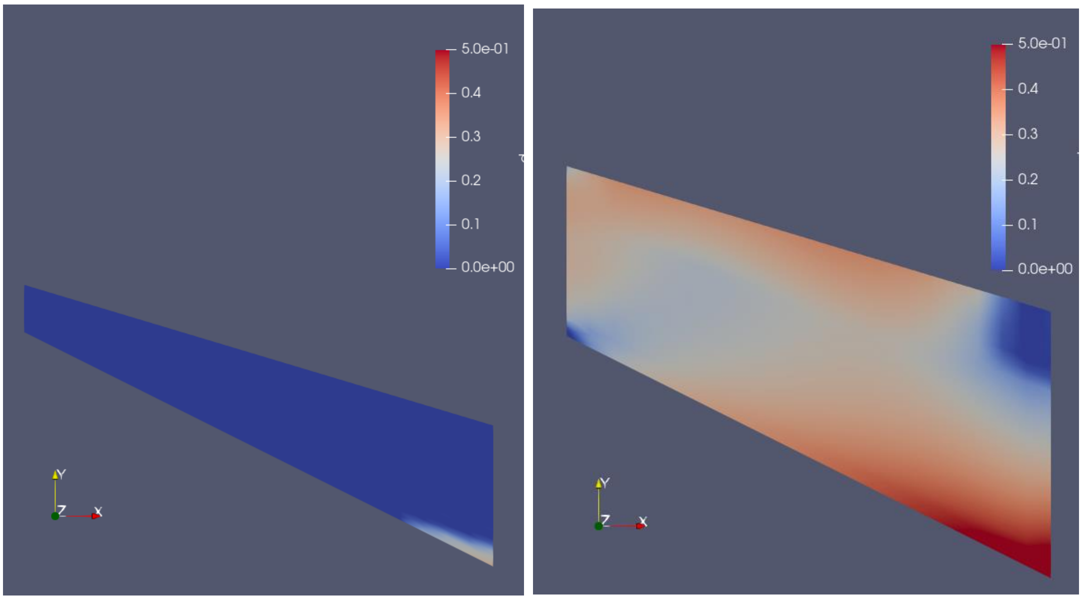


Figure 21: Damage color-coded for two different back heights, 10m on the left and 35m on the right (value range 0 (blue)- 0.5 (red)) at time step 2

3.2.2.3.2. Break-off pattern and rate

The thicker glacier breaks off completely were fast, with the break-off rate not slowing down until the whole glacier is gone. The thinner glacier breaks off slowly and gradually similarly as seen before with model results where parameters reduced damage evolution. Figure 22 shows the plot of the break-off rate for the models with a back height of 10m, 15m, and 20m. It is very clear how important the back height, respectively the mean thickness is for the stability of the glacier, as the differences in break-off rate are tremendous for the small range of back height values.

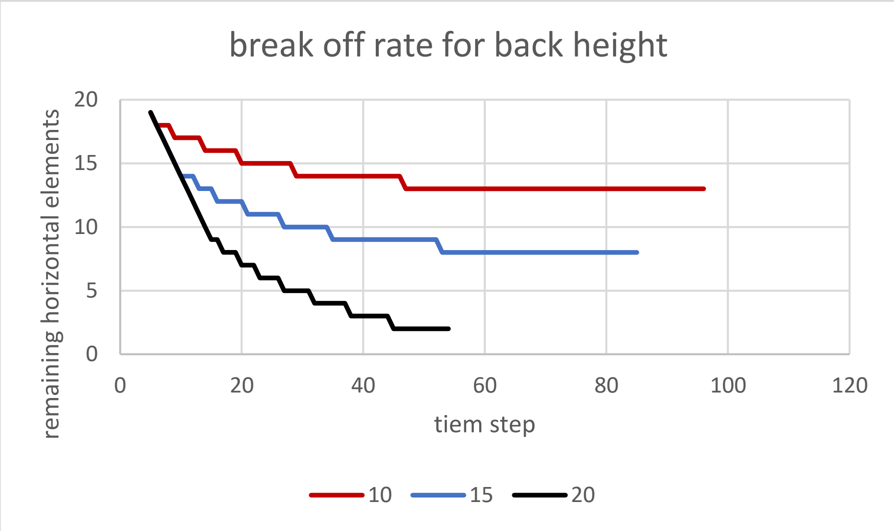


Figure 22: Plot of the break-off rates for the different back heights (10 in red, 15 in blue, and 20m in black) for the standard glacier geometry

3.2.2.3.3. Additional observations

With an increase in the mean height, the ice flow velocities increase strongly as well. Both the pattern and the magnitude of ice flow velocities change significantly. While up to now we have only seen high flow velocities at the terminus, for the thick glacier the zone of higher velocities spans back to about half of the glacier’s length.

3.2.2.3.4. Stress analysis

The analysis of basal stresses for the changes in back height shows that shear stress, horizontal stress, and vertical stress increase with increasing back height. This is what we would expect as with the mean thickness the overburden pressure increases. This explains with the damage evolves faster with a higher mean thickness. The result of the stress analysis is presented in figure 23.

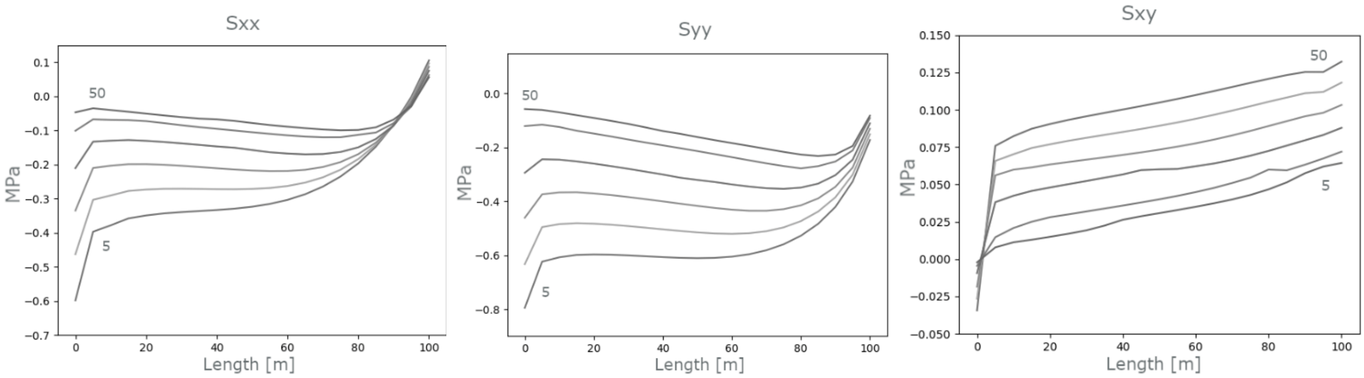


Figure 18: Results of the stress analysis for the different back heights (5m, 10m, 20m, 30m, 40m, 50m). The right is sxx (horizontal stress) in the middle syy(vertical stress) and on the right sxy (shear stress)

3.3. Critical stable geometry investigation

To further explore critical stable geometries, we again chose a lower back height of 10m. For the same parameters, we run the model for three different lengths of the glacier. The results of the study of the length parameter have shown that for a fixed geometry and parameters, glaciers of different lengths will converge on a critical stable remaining length. We will take a closer look at the critical stable geometry as we perform the same modeling with varying a parameter and comparing the result to the reference from the length parameter study. As the back height has proven to be important a thinner glacier was chosen.

Figure 24 shows the reference glacier model result from the length parameter study on the left and the thinner glacier results on the right for the time step 199. The results have shown again that the glaciers of different lengths converge on a critical stable remaining geometry. In comparison to the reference glacier, the thinner glaciers' critical stable geometry is longer. Now, this was to be expected, The background knowledge gained from conducting the parameter studies explains that for the lower mean thickness the break-off rate is decreased due to the lower stresses, the damage evolves slower and only in a small concentrated damage zone.

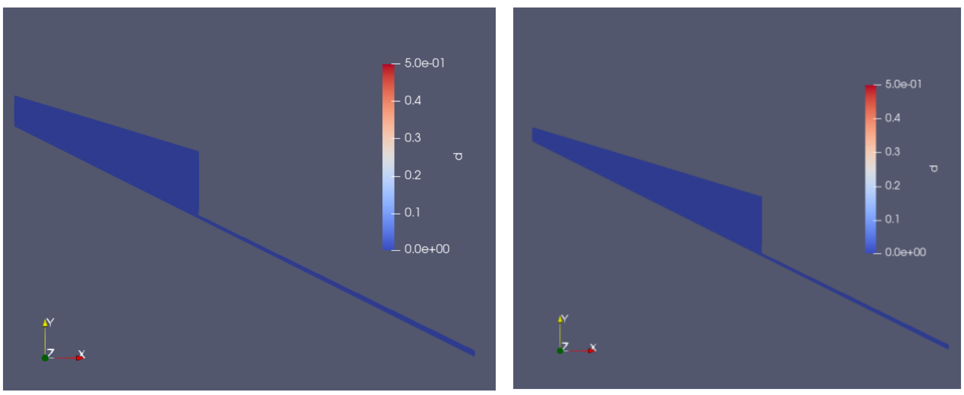


Figure 19: Two different critical stable geometries for a thicker glacier H0=10m on the left and thinner glacier H0=5m on the left at time step 199

3.4. Representation of a real hanging Glacier

As the last step of this modeling approach, we want to introduce a simplified geometry of a real cold-based hanging glacier. The Grande Jorasses cold-based ramp glacier was chosen. With this step, we are trying to see how well the model adapts to real-world glaciers. It is also an approach to the verification of the model parameters.

Treating the Grande Jorasses geometry as the critical stable geometry, the goal is to adjust the model parameters, in a way that a supercritical length glacier will collapse and settle at the desired critical stable geometry. The parameter study will now help to choose how to adjust the parameters to find an optimal solution. Because the geometry is set, we can only adjust non-geometry parameters. As the parameter study has shown the Glen's flow factor A does not significantly influence the stability, however, we did include it in this assessment and, as we had already stated, found that the facto A has a marginal influence in this steep glacier geometry when modeled with a low damage rate. The difference is so slight that when compared side to side it is very hard to spot if one cannot directly switch from one to the other. The damage value is however marginally lower for the lower A factor.

With that out of the way the remaining two parameters are the damage rate and the healing term. We decided to model different damage rates for two coupled healing terms. The standard low healing with hsig 5 and he0 0.5 and a medium healing at hsig=15 and he0=1.5.

The result for both healing terms one damage rate delivered the desired result. For the low healing term, the critical stable geometry was reached with a damage rate of 30. With the medium healing term, it took a damage rate of 40 to reach the desired result. This would suggest that in fact healing is needed for an accurate representation of hanging glaciers. Secondly, it would also mean that the damage rate of 65 is not suitable for hanging glaciers and needs to be recalibrated. Figure 25 shows the output of the model run using the medium healing term and a damage rate of 40.

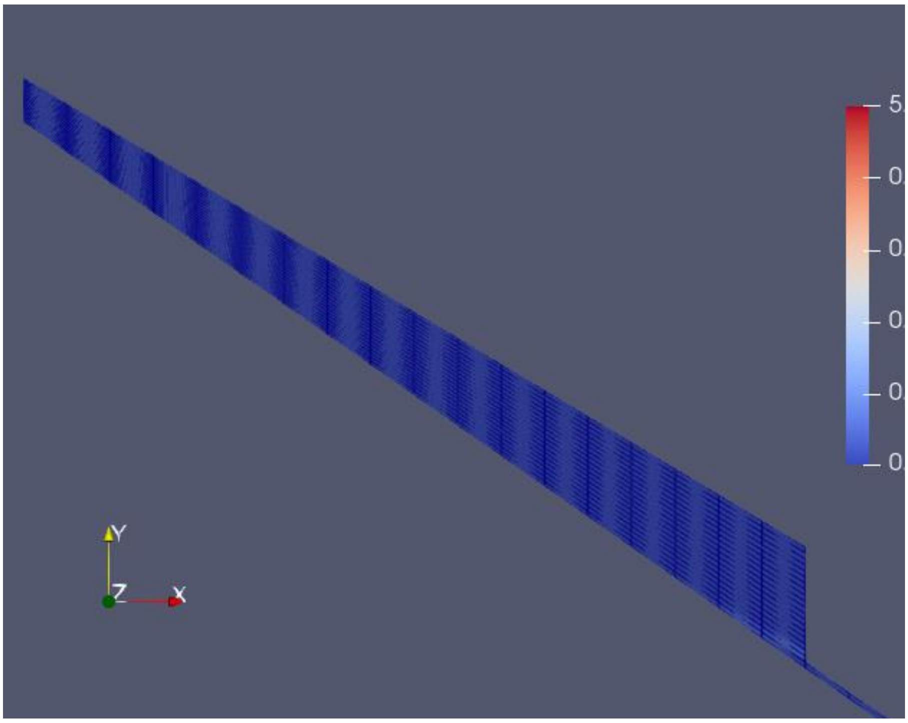


Figure 20: Critical stable geometry of the simplified Grande Jorasses matching the originally derived simplified critical stable geometry for the glacier at time step 199, this was achieved by combining B=40 and hsig=15/he0=1.5

4. Discussion

The focus of this thesis lies in modeling cold base on hanging glaciers and we aim to obtain a better understanding of the dynamics of hanging glaciers through modeling. We posed two main questions; What are the sensitivities of the model and of hanging glaciers to different parameters? And can it be estimated how good the mode represents real hanging glaciers?

4.1. General model assessment

We started with a general assessment of the model, this was great to take a closer look at how the model works and what it does, before heading into the parameter studies. With this first step, we could ensure that the model worked correctly. We made a comparison of how the model performs based on what we would expect to happen for this geometry.

4.1.1. Damage and break-off pattern

Looking at the distribution of damage the results were as we expected them. The main damage formed at the terminus base where would expect the shear stress maximum, as described by Pralong & Funk (Pralong & Funk, 2006). It was also very clear that slab fracture was in process, as elements broke off at the base. Again, for a ramp glacier like we modeled, we would certainly expect slab fracturing (Pralong & Funk, 2006). The surface damage zone seems interesting as well and reminds us of a surface crevasse zone that would fit the velocity acceleration towards the terminus and the resulting extension.

4.1.2. Velocity

The distribution of ice flow velocity was also just as expected with an acceleration towards the terminus, which was strongest at the surface and decreased with depth.

This proved that we were on the right track and ready to start investigating the sensitivities of the model to different parameters

4.2. Parameter study

With the parameter study, we were able to answer our first main question. We discovered distinct sensitivities of the model and therefore modeled glaciers on different parameters. The qualitative assessments provide a nice overview of the reaction of the main processes to the variation of parameters. In most cases, we could already determine or at least estimate the overall effects of the parameter changes. The analysis of bed stresses delivered good insight on why damage developed how it does and how it changes when modifying the geometry parameters. The break-off rate we introduced was a successful tool for comparing the stability or instability of the model results. In our opinion, it also provides a nice inter-comparability of the different parameter variations that we did during the parameter study.

4.3. Critical stable geometry approach

We wanted to base the verification approach on the critical stable geometry which focuses on the stable geometry threshold. For this reason, we briefly explored the critical stable geometry for different parameters, to assess the validity of the concept. It proved that the model does converge on one critical stable geometry and the reaction to variation of the parameter showed the expected result. For this reason, we decided to go further with this approach and use it in the example of the Grande Jorasses glacier geometry.

4.4. Real glacier representation

The result from the simplified Grande Jorasses model suggested that either the damage rate was too high, the healing plays an important role, or a combination of the two. There are however problems in deciding which option is correct.

On one side the used damage rate of 65 was taken from the study that developed the model and focuses on ocean terminating calving glaciers. The damage rate has been calibrated to fit those types of glaciers. While der are similarities between wet calving and dry calving glaciers, are as many vast differences as possible.

On the other side is the concept of the healing term. While Pralong & Funk clearly stated that healing is an important process, they also addressed that the current concepts of healing parametrization are unphysical and do not have a thermodynamic basis (Pralong & Funk, 2005). This makes it very hard to say how, or if at all, the way that healing is modeled, and scale is connected to reality

This uncertainty leads us to the conclusion that, that we cannot definitively say which option is correct and to what degree the factors play a role. Based on the results of the whole modeling project we would however guess that it is a combination of the two parameters.

Nevertheless, the results open up a new question that could be taken up in future research.

4.5. Lessons for improvement

For future projects, there are some lessons to be learned as in retrospect there is some room for improvement. In the beginning, it was underestimated that it would take some time to get used to the new software environment Getting into the "modeling mindset" was sometimes also difficult for someone coming from a rather fieldwork project cantered background. It was a challenge. A big point to be addressed would also be efficiency. Looking back, it would have made sense to get quicker into the parameter study, to build up more of a focus. This would probably also have meant to have python pre-loop earlier which cut down time invested into modeling by a lot.

4.6. Outlook

Possible future research should pick up where we left off. There were some points that we could not go further into, which were not in the center of our focus, but which would be interesting and important. With this thesis, we laid a foundation upon which future projects could build.

4.6.1. parameter combinations

Let's start with a smaller point. While we focus on the sensitivities of the model to parameter changes, an interesting point would be to do more combinations of the parameter to explore their interaction. This was just not in our focus as the interaction of parameters is from our perspective secondary to the true parameter sensitivity.

4.6.2. Accumulation

The major point which should be explored is accumulation. Accumulation would introduce another level of dynamics to the modeling of hanging glaciers. But it also adds to the complexity of the modeling. Accumulation would be interesting because the glacier would also be able to advance. This could be interesting to explore the build of a hanging glacier until the critical geometry with the subsequent break-off.

4.6.3. Polythermal glaciers

A next step could also be to switch to polythermal hanging glaciers and investigate what is different. This would be important as for many glaciers the basal conditions are only available as an estimate (Pralong & Funk, 2006). In the far future, the model might even be used to determine what estimate is more accurate by modeling the glacier once for both conditions and comparing which behavior comes closer to reality.

4.6.4. Verification

While we did offer an approach to comparing the modeled glacier to its real counterpart, the approach is very simplistic. Out of constraints we also only performed this analysis for one specific glacier. To really be able to suggest that healing is as important or that the damage rate is too high, we would need to start comparisons for multiple different glaciers. This would however far exceed the aim and frame of this thesis, as one could write a whole thesis on finding the perfect fit of parameters.

4.6.5. Decrease simplification

We have only modeled idealized glaciers with very simple geometries. In the future, the model could be adapted to complex bed geometries to more accurately represent real glaciers. For the assessment of the model as performed by us, it was however better to keep the model simple, as it allowed for clearer deductions based on the results.

5. Conclusion

We conclude that our research was successful as we could answer our two main questions, achieving the goals set at the beginning of this project. We did manage to explore the sensitivities of the model to different parameter changes and clearly determine their effect on the stability of a glacier. Decreasing the damage rate and increasing the healing term improves the stability, the Glen's flow factor did not have any significant effect on the stability of the modeled glacier. Increasing the length and back height results in a loss of stability. Increasing the paired slope and slope difference resulted in decreased stability. The model has shown, that for a fixed set of parameters glaciers varying only length will converge on a critical stable geometry. This critical stable geometry reacts as expected to changes in parameters, for example decreasing the back height and therefore the mean thickness results in a longer critical stable geometry. Modeling the simplified Grande Jorasses glacier, we showed that healing could be important and a key parameter. The other suggestion would be that the damage rate used is not suitable for cold-based hanging glaciers. While we are not confident in saying that either of these suggestions is fact, it is an important point to be raised and a foothold for future research.

There was also a personal gain not only knowledge of hanging glaciers, but also in experience with challenging new software that led to some struggle which was however successfully overcome and maybe most importantly a gain in experience in modeling, since this was the first true modeling project. Looking back there are a lot of small things that could have been made better and especially more efficiently but it was part of the learning process.

6. References

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6.1. Additional background reading

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6.2. Table of figures

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